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FOUR YEAR B.A./B.Sc./B.Com./B.B.A/B.C.A. (Honors) DEGREE EXAMINATION, NOVEMBER/DECEMBER 2025.

FIFTH SEMESTER

Mathematics

Paper 12: LINEAR ALGEBRA

(w.e.f. 2023-24 Regulations)

Time : Three hours | Maximum : 70 marks

(No additional sheet will be supplied)

SECTION A — (5 × 4 = 20 marks)

Answer any FIVE questions.

1. Show that $\{(a,b,c); a,b,c \in R\}$ and $a-3b+4c=0\}$ is a subspace of $V_3(R)$.

2. Show that the system of three vectors (1, 2, 0) , (0, 3,1), (–1, 0, 1) of $V_3(Q)$ is linearly independent.

3. Show that the vectors (1, 0, 0), (1, 1, 0), (1, 1,1) forms a basis of $C^2(C)$.

4. If W is subspace of $V_4(\mathbb{R})$ generated by the vectors. $(1,-2,5,3)$,$(2,3,1,-4)$ and $(3,8,-3,-5)$ then find a basis of W and its dimension .

5. Find $T(x,y,z)$ where $T: R^2 \to R$ is defined by $T(1,1,1)=3$, $T(0,1,-2)=1$, $T(0,0,1)=-2$.

6. Find the kernel of the linear transformation $T: R^2 \to R^2$ defined by $T(1,0)=(1,1)$ and $T(0,1)=(-1,2)$.

7. Find the characteristic roots of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

8. If $A=\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ then express A^{-1} as a linear polynomial in A by using Cayley- Hamilton theorem.

9. State and prove parallelogram law.

10. Show that $\{(1,0,0),(0,1,0),(0,0,1)\}$ is an orthonormal set in the inner product space V_3 $(\mathbb{R})$.

SECTION B — (5 × 10 = 50 marks)

Answer ALL the following questions.

11. The necessary and sufficient condition for a non-empty subset W of a vector space $V(F)$ to be a subspace of V is that $a, b \in F, \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$.

Or

12. Let W_1 and W_2 be two Subspaces of a Vector Space $V(F)$. Then $W_1 \cup W_2$ is a sub space of $V(F)$ iff $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

13. Prove that 'Every finite dimensional vector space has a basis'.

Or

14. If W_1 and W_2 are two subspaces of a finite dimensional vector space $V(F)$ then prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

15. State and prove rank nullity theorem.

Or

16. Show that the mapping $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + 2y, y - z\ x + 2z)$ is a linear transformation. Find the rank and nullity and verify the formula $rankT + nullityT = \dim R^3$.

17. Find the characteristic roots and characteristic vectors of the matrix $A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$.

Or

18. State and prove Cayley Hamilton Theorem.

19. State and prove Bessel's Inequality theorem.

Or

20. State and prove Gram-Schmidt orthogonalization process.
